MATH 347: FUNDAMENTAL MATHEMATICS, FALL 2015

PRACTICE PROBLEMS FOR MITERM 1

1. Prove that for all sets A, B

(a) $A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$.

(b) $(A \cup B) - B = A - B$.

2. Prove or give a counter-example:

(a) For every function $f : X \to Y$ and every $B \subseteq Y$, $I_f(B)^c = I_f(B^c)$.

(b) For every function $f : X \to Y$ and every $A \subseteq X$, $f(A)^c = f(A^c)$.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined as follows: for $x \in \mathbb{R}$,

$$f(x) = \begin{cases} x^2 & \text{if } x \leq -1 \\ |x| & \text{if } -1 \leq x \leq 1 \\ x & x \geq 0 \end{cases}.$$

Is f a well-defined function? Justify your answer.

4. (a) Let $g : \mathbb{R} \to \mathbb{R}$ be the absolute value function, i.e. $g(x) = |x|$ for each $x \in \mathbb{R}$. What is $I_g\Big(g\big([-1, 0)\big)\Big)$?

(b) In general, for an arbitrary function $f : X \to Y$ and $A \subseteq X$, what is the relation between A and $I_f\big(f(A)\big)$? Prove your answer.

5. Recall the definition of linear independence for points (vectors) in $\mathbb{R}^n$.

Definition. Vectors $\vec{v}_1, \vec{v}_2 ..., \vec{v}_k \in \mathbb{R}^n$ are called *linearly independent* if

$$\forall a_1, a_2, ..., a_k \in \mathbb{R}\Big[a_1\vec{v}_1 + a_2\vec{v}_2 + ... + a_k\vec{v}_k = \vec{0} \implies \big(\forall i \leq k, a_i = 0\big)\Big].$$

Vectors $\vec{v}_1, \vec{v}_2 ..., \vec{v}_k \in \mathbb{R}^n$ are said to be *linearly dependent* if they are not linearly independent.

(a) Write out explicitly what it means for vectors $\vec{v}_1, \vec{v}_2 ..., \vec{v}_k \in \mathbb{R}^n$ to be linearly dependent. The only negation sign/word in your sentence should be negating equality $\neq$.

(b) Are the vectors $(1, 1)$ and $(1, 0)$ linearly independent? Prove your answer.

(c) Are the vectors $(1, 0, 0)$, $(0, 1, 1)$ and $(1, 1, 1)$ linearly independent? Prove your answer.

6. Consider the sequence $(x_n)_{n \in \mathbb{N}}$, where $x_n = \frac{(-1)^n}{n^2}$. Determine whether the following are true or false, and prove your answer in either case.

(a) $\forall \varepsilon > 0\ \exists N \in \mathbb{N}\ \forall n \geq N\ |x_n| < \varepsilon$.

(b) $\exists N \in \mathbb{N}\ \forall n \geq N\ \forall \varepsilon > 0\ |x_n| < \varepsilon$.

7. Do the induction/strong induction problems in HW5. Also, review the induction problems in HW4.

8. In general, it would really help to review HW1–HW4.